



# Dynamics of Networks

Online Social Networks Analysis and Mining

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# Where next?

Two kind of dynamics:

- **Dynamics of Networks**  
(topological perturbations)
- **Dynamics on Networks**  
(diffusive phenomena: epidemics, opinion dynamics...)

Of course they can happen at the same time...

Dynamics of Networks	Dynamics on Networks	Mixed Dynamics
<p><b>Assumption:</b> Topology evolution is faster than diffusive processes unfolding (if any)</p> <p><b>Applications:</b></p> <ul style="list-style-type: none"><li>- Link Prediction</li><li>- Dynamic Community Discovery</li><li>- ...</li></ul>	<p><b>Assumption:</b> Diffusive processes unfolding is faster than topology evolution (if any)</p> <p><b>Applications:</b></p> <ul style="list-style-type: none"><li>- Epidemic spreading</li><li>- Opinion Dynamics</li><li>- ...</li></ul>	<p><b>Assumption:</b> Diffusive processes unfolding and topology evolution have comparable rates</p> <p><b>Applications:</b></p> <ul style="list-style-type: none"><li>- Diffusion shape topology</li><li>- Topology shape diffusion</li><li>- Feedback loops</li></ul>

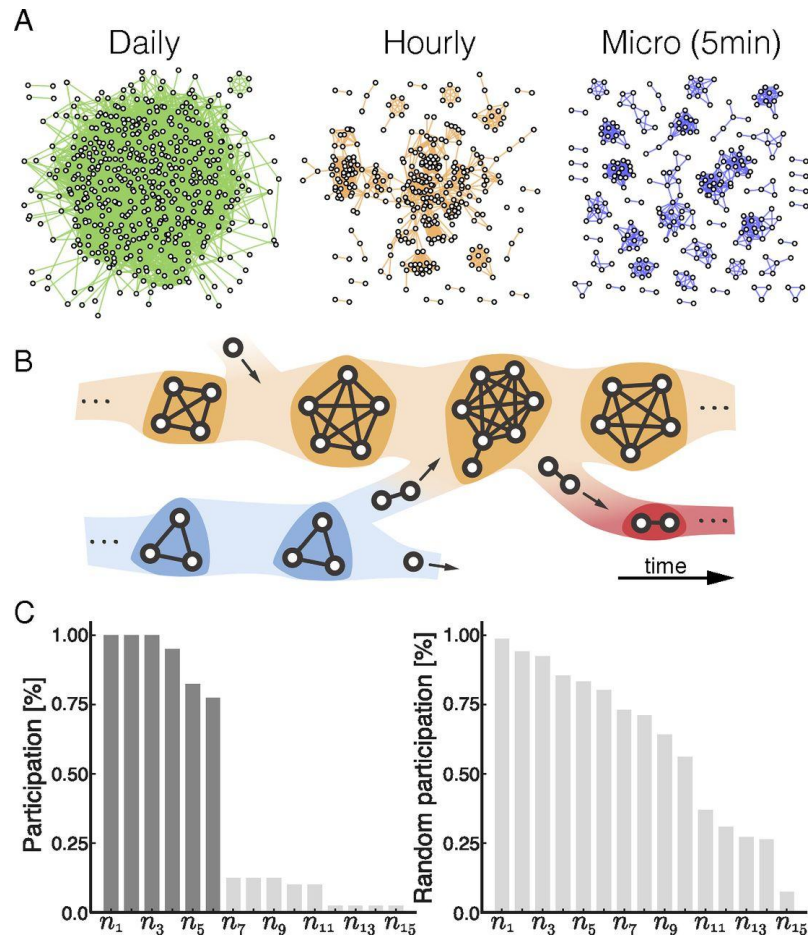
# Representing Dynamic Topologies

A brief Introduction

# Why bother of time?

Most real world networks are **dynamic**

- Facebook friendship
  - People joining/leaving
  - Friend/Unfriend
- Twitter mention network
  - Each mention has a timestamp
  - Aggregated every day/month/year => still dynamic
- World Wide Web
- Urban networks
- Protein-protein interactions
- Brain networks
- ...



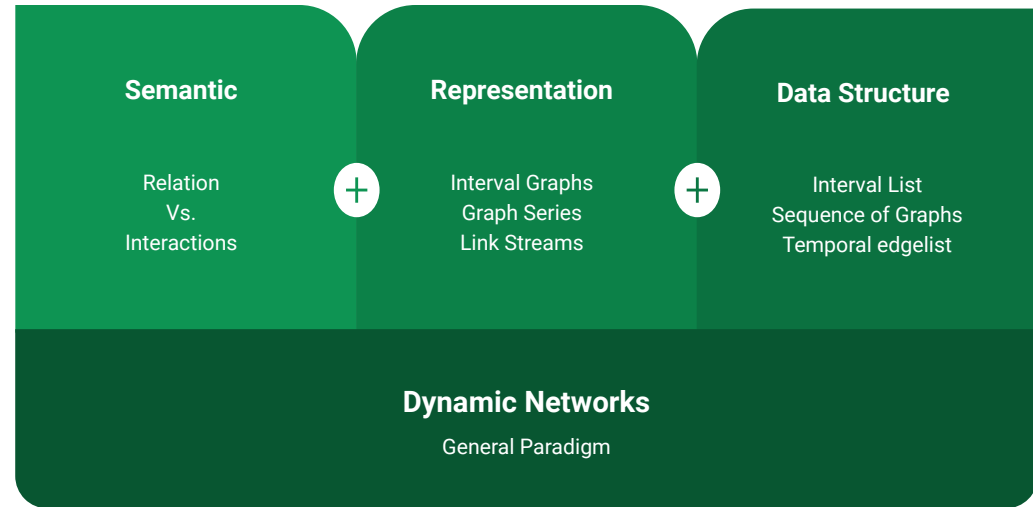
# Evolving Topologies

- Nodes can appear/disappear
- Edges can appear/disappear
- Nature of relations can change

How to **represent** those changes?

How to **manipulate** dynamic networks?

Three different levels of abstraction



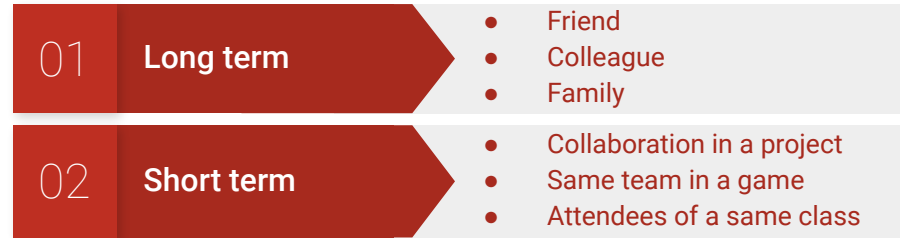
# Relations Vs. Interactions

Topological perturbations may have different **temporal scales** depending on their intrinsic **semantic value**.

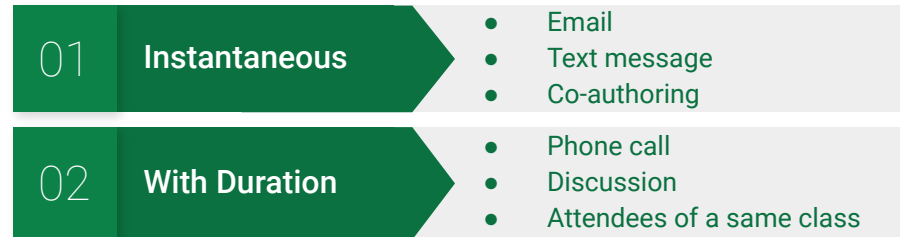
Two families:

- Relations (stable ties)
- Interactions (unstable ties)

## Relations



## Interactions



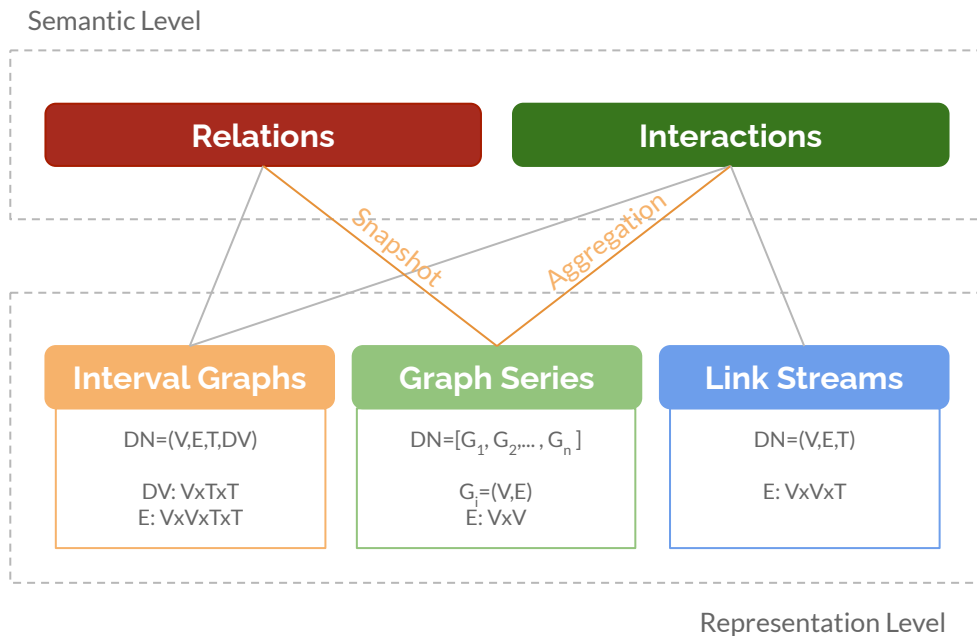
# Semantics and how to represent them

## Relations

The graph is more and more stable, until most observations are completely similar to previous/later ones (frequency faster than change rate)

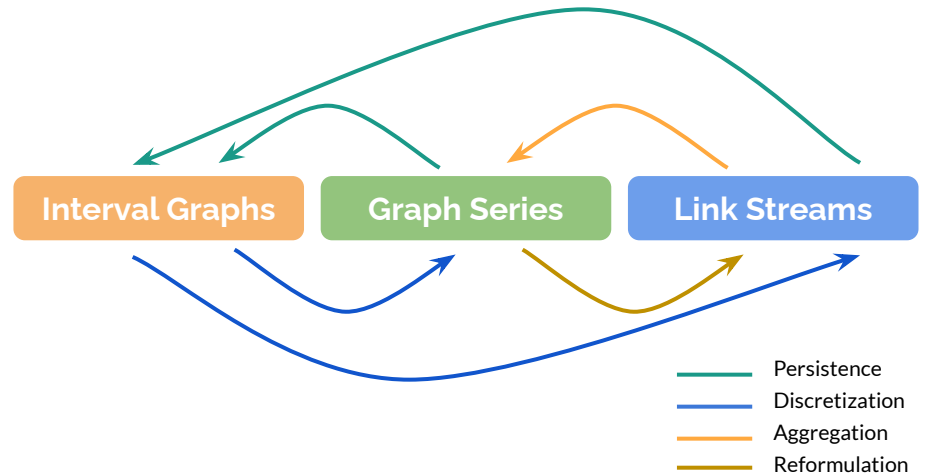
## Interactions

The graph is less and less stable, until each observation is a graph in itself, thus completely different from previous/later ones (frequency faster than observed events rate)



# Changing Representation

Alternative representations can be, to some extent, **converted** among them by applying appropriate data **transformations**

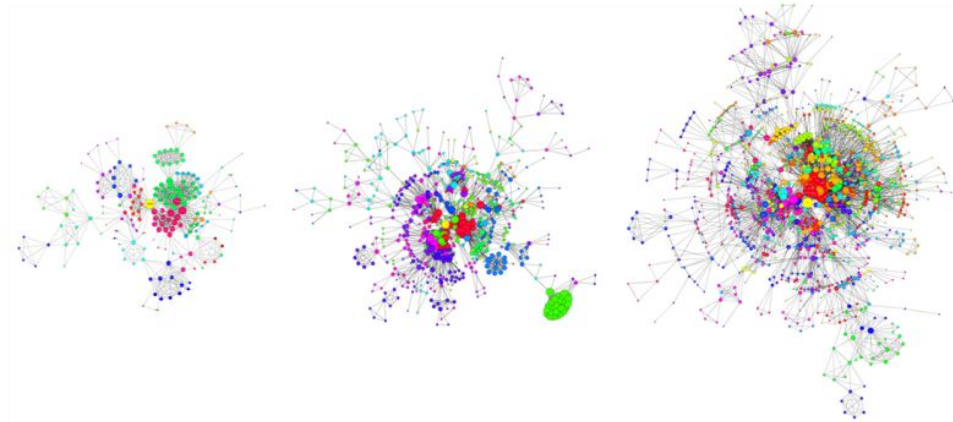




# Unstable Snapshots

The evolution is represented as a series of a few snapshots

- Many changes between snapshots  
(Cannot be visualized as a “movie”)
- Each snapshot can be studied as a static graph
- Evolution of node properties can be studied “independently”  
(e.g., node  $i$  had low centrality in snapshot  $t$  and high centrality in snapshot  $t+n$ )



# Stable Network

Edges change (relatively) slowly

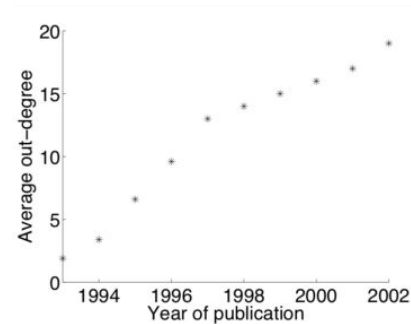
The network is well defined at any  $t$

- Temporal network: nodes/edges described by (long lasting) intervals
- Enough snapshots to track nodes

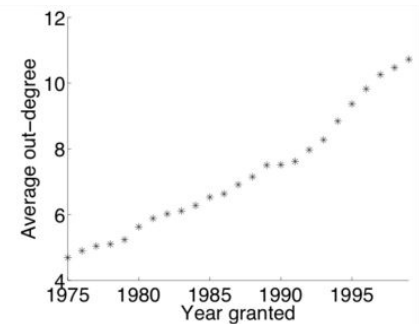
A static analysis at every (relevant)  $t$  gives a dynamic vision

No formal distinction with previous case (higher observation frequency)

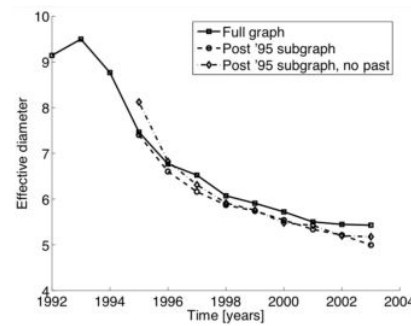
Properties can be analyzed as time series



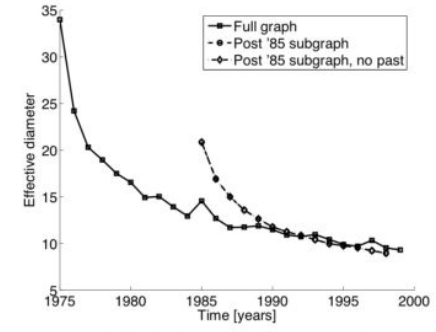
(a) arXiv



(b) Patents



(a) arXiv citation graph



(c) Patents citation graph

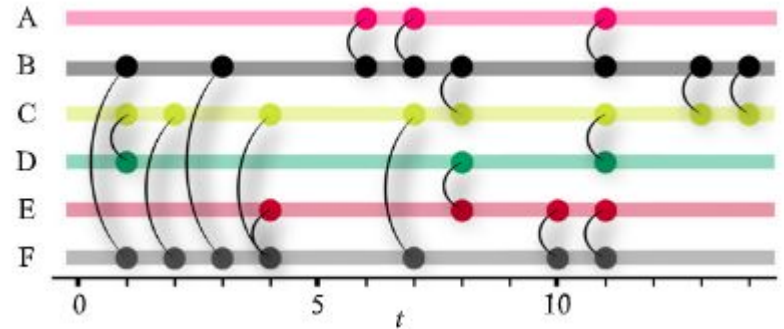
# Unstable Temporal Network

The network at a given  $t$  is not meaningful

How to analyze such a network?

Until recently, network was transformed using aggregation/sliding windows

- Information loss
- How to choose a proper aggregation window size?



Unstable

Temporal Nets

# Stream Graph

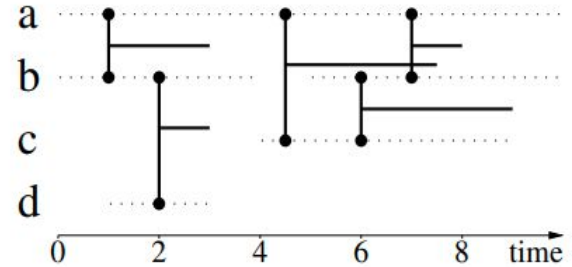
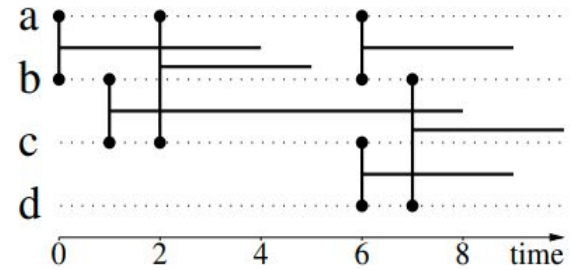
$$S = (T, V, W, E)$$

T: Possible Time

V: vertices

W: Vertices presence in time, pairs like  $(v, t)$

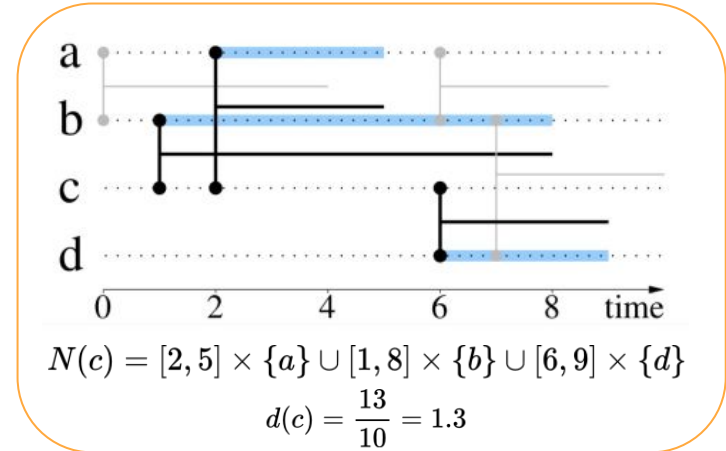
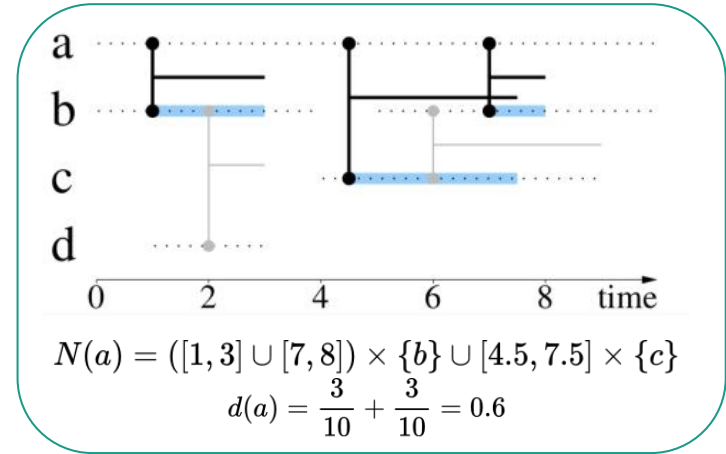
E: Edges presence in time



Latapy, M., Viard, T., & Magnien, C. (2018). Stream graphs and link streams for the modeling of interactions over time. *Social Network Analysis and Mining*, 8(1), 61.

# Indices

Number of nodes	$n = \sum_{v \in V} n_v = \frac{ W }{ T }$
Number of edges	$m = \sum_{uv \in V \otimes V} m_{uv} = \frac{ E }{ T }$
Neighbors of a node	$N(v) = \{(t, u), (t, uv) \in E\}$
Degree of a node	$d(v) = \frac{ N(v) }{ T } = \sum_{u \in V} \frac{ T_{uv} }{ T }$

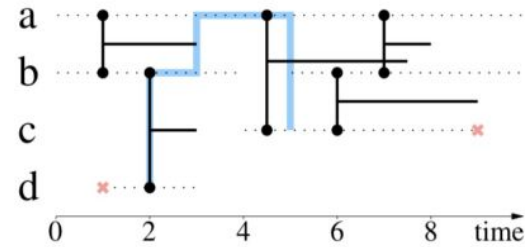


Stream Graph

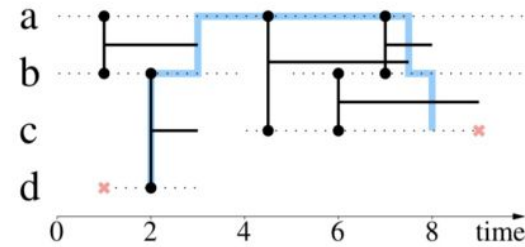
# Paths and Distances

A path in a stream graphs

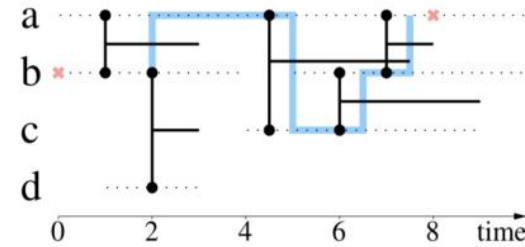
- Starts at a node and at a timestamp
- Ends at a node and at a timestamp
- Has a length (number of hops)
- Has a duration (duration from leaving node to reaching node)



Path: (d,1)(c,9)  
Length: 3  
Duration: 3



Path: (d,2)(c,8)  
Length: 4  
Duration: 6



Path: (b,0)(a,8)  
Length: 4  
Duration: 5

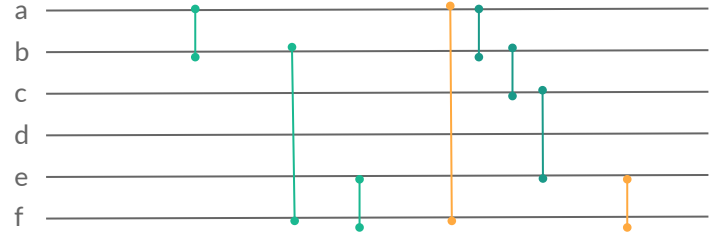
Stream Graph

# Paths and Distances

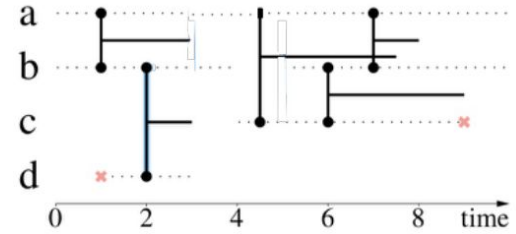
Several types of shortest paths in Stream graphs:

Type	Description
Shortest path	Minimal length
Fastest path	Minimal Duration
Foremost path	First to reach
Fastest shortest paths	Minimum duration among minimum length
Shortest fastest paths	Minimum length among minimum duration

From a to e (Foremost, Fastest, Shortest)



From (1,d) to (9,c)



- Shortest path (2.5, d, b) (3, b, a) (7, a, c)
- Fastest path (3, d, b) (3, b, a) (4.5, a, c)
- Foremost path (2, d, b) (2, b, a) (4.5, a, c)
- Fastest shortest path (3, d, b) (3, b, a) (4.5, a, c)
- Shortest Fastest path from (3, d, b) (3, b, a) (4.5, a, c)

# Community Detection in Dynamic Networks

Time flies like an arrow; fruit flies like a banana



# Communities In Dynamic Networks

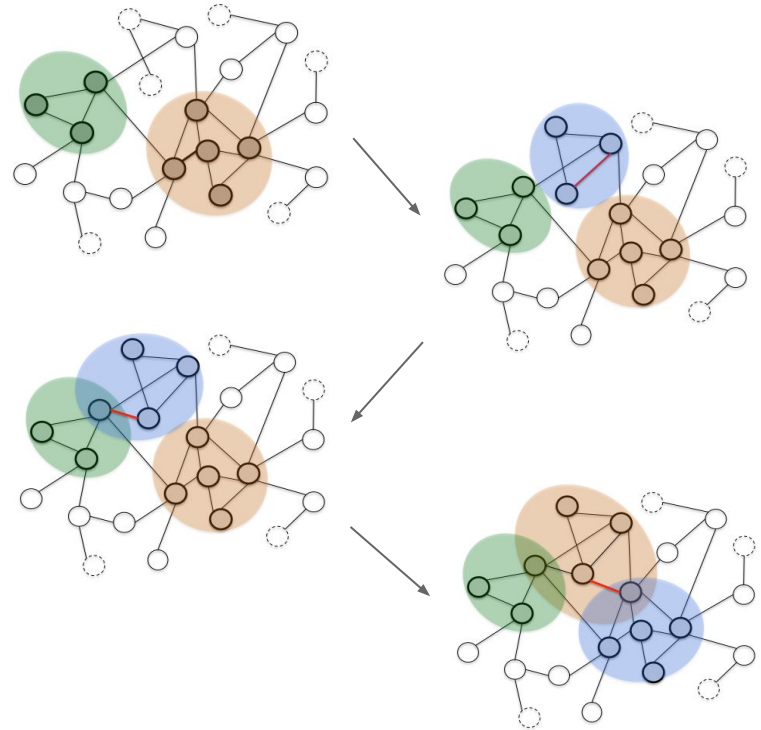
Networks change with time...

- Nodes appear and vanish
- Edges appear and vanish

...communities must change too!

DCD:

identify/track changes in community structure



Cazabet, Remy, and Giulio Rossetti. "Challenges in community discovery on temporal networks." *Temporal Network Theory*. Springer, Cham, 2019. 181-197.

A Novel Problem:

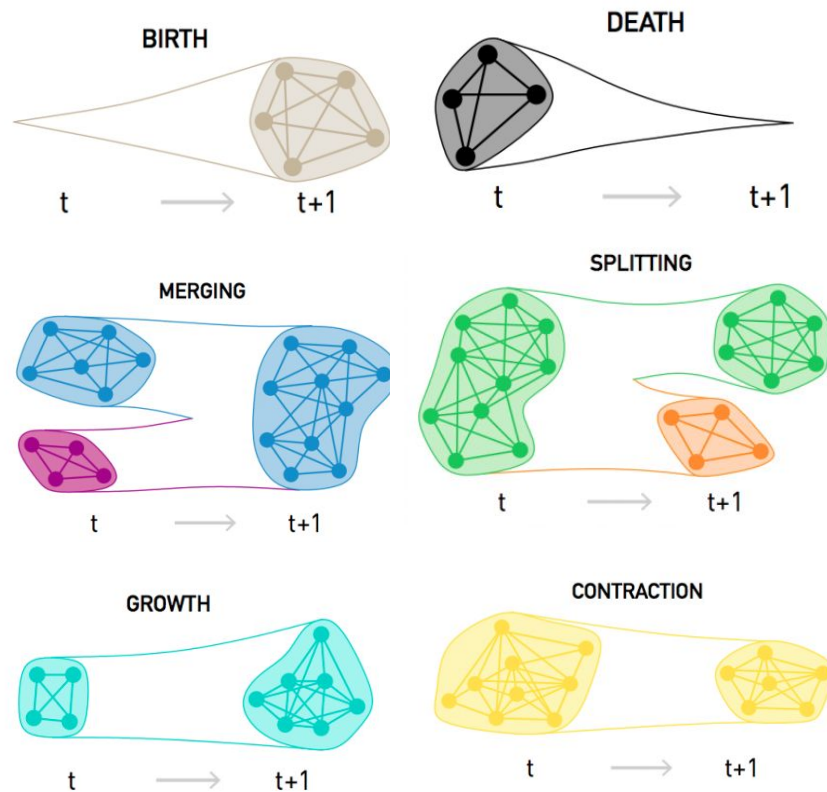
# Community life-cycle tracking

As time goes by the **rising** of novel nodes and edges (as well as the **vanishing** of old ones) led to network perturbations

Communities can be deeply affected by such changes

Three main strategies:

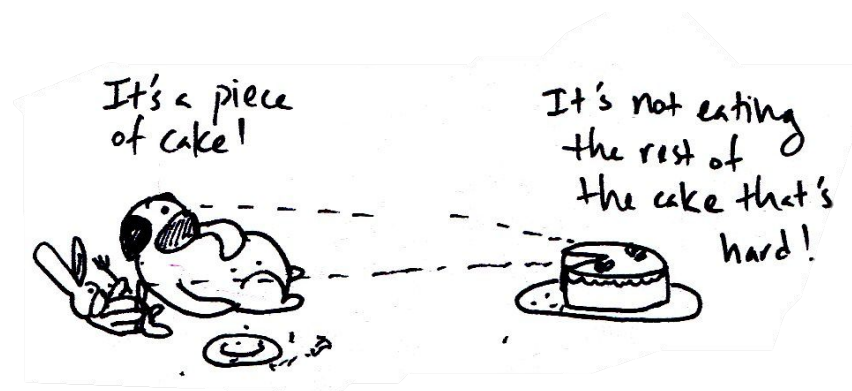
- Identify & Match
- Informed Iterative algorithms
- Stable Identification



The Optimist:

# “Ok, It’s a piece of cake!”

1. Find communities at each network observation (using a static algorithm)
2. Match communities across consecutive network observations
3. Observe differences



Two major issues:

- Community Smoothing
- Theseus' Ship Paradox

# Community Smoothness

Communities are arbitrarily defined  
(same issue of static CD)

Most “efficient” algorithms are stochastic

- Change in communities might be due to **structural changes** OR to **arbitrary choices** of the algorithm
- The same algorithm ran twice on the same graph *might yield* different results

Desiderata:

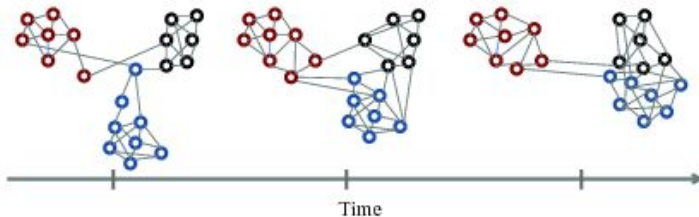
- a “simple” (parsimonious) model
- a trade-off between quality and simplicity (smoothness)

## No Smoothness:

Partition at each  $t$  should be the same as found by a static algorithm

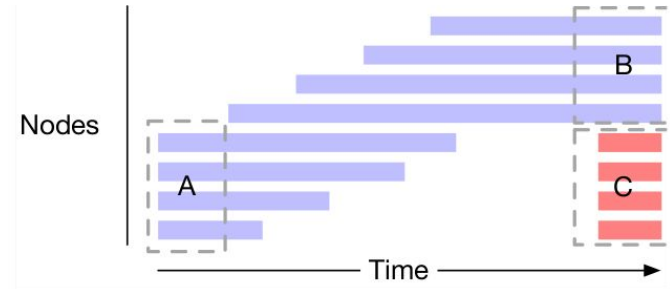
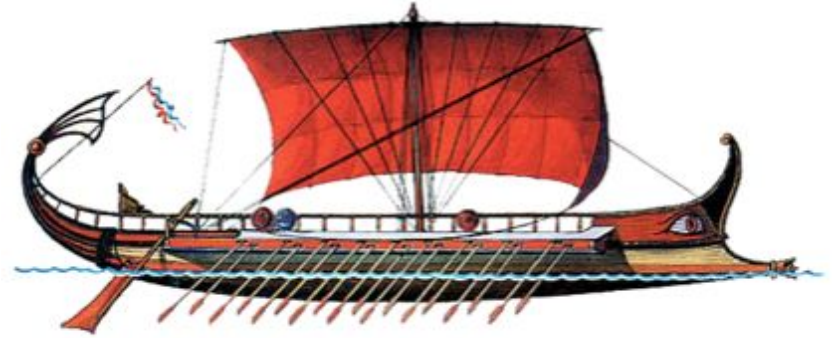
## Smoothness:

Partition at  $t$  is a trade-off between “good” communities for the graph at  $t$  and similarity with partitions at different times



# Theseus' Ship Paradox

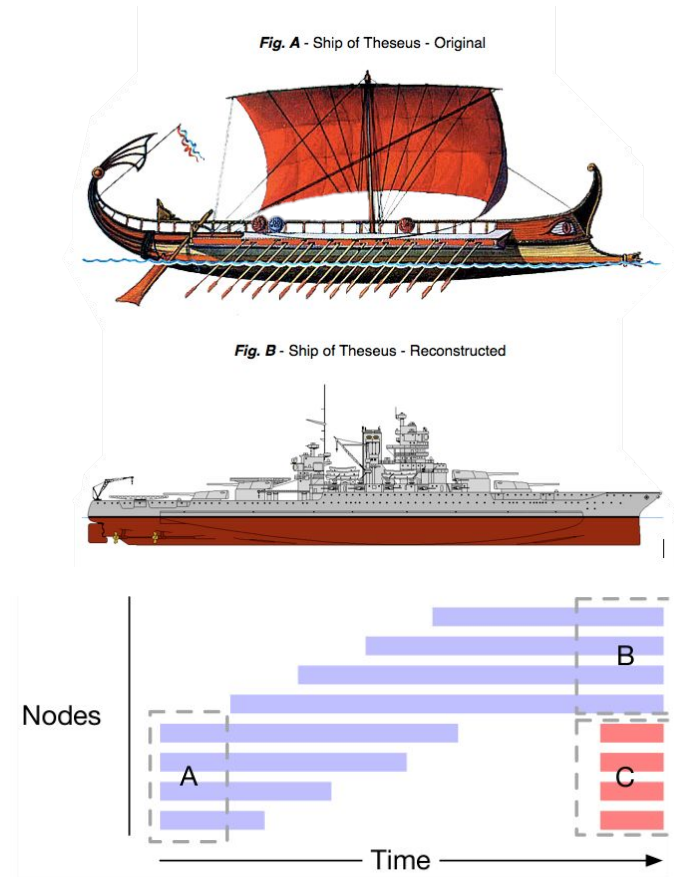
- I. Theseus killed the Minotaur in Crete and came back to Athens on his boat
- II. His boat was conserved as memory during a very long time
- III. The boat was deteriorating, so pieces of it were gradually replaced.
- IV. Until one day, all original parts were replaced



# Theseus' Ship Paradox

- A. Is this ship still the same as Theseus boat ?
- B. If another boat was built using all pieces of the original boat, which one would be the "real" Theseus boat ?

Community evolution/identity is an arbitrary concept



# Community Detection in Dynamic Networks

A taxonomy

# DCD Algorithms Taxonomy

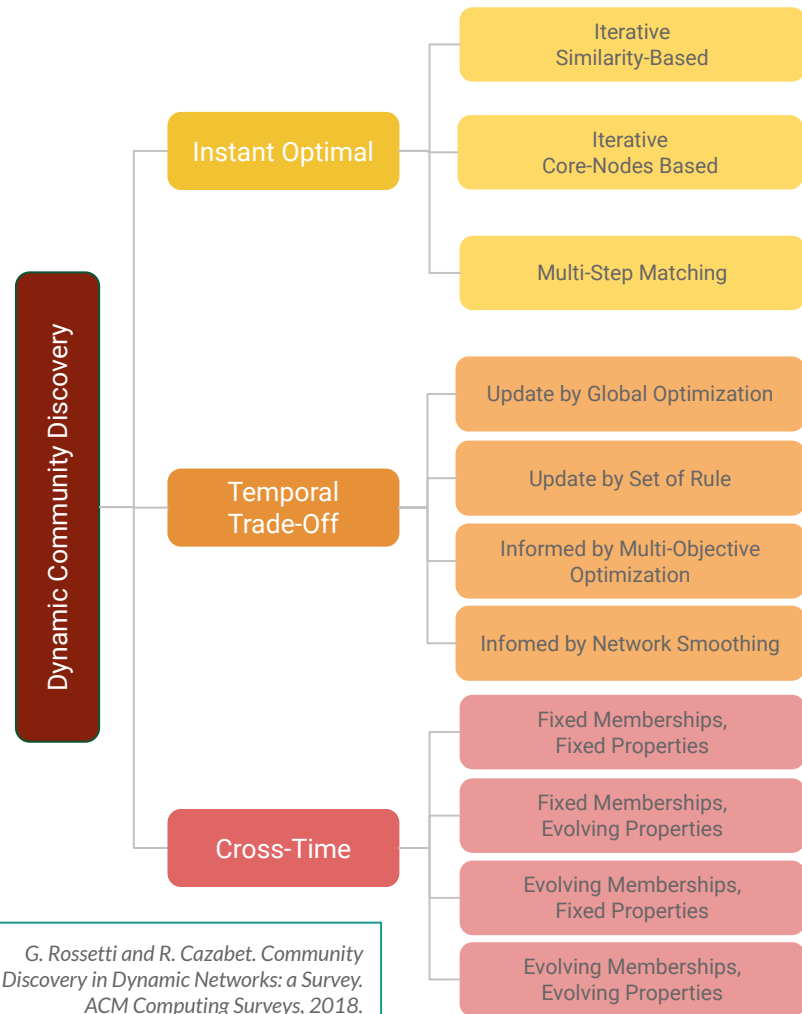
Hierarchical categorization

First Level:

Increasing degree of smoothness (*none* -> *complete*)

Second Level:

Algorithmic Approach (*how to deal with Theseus*)



G. Rossetti and R. Cazabet. Community Discovery in Dynamic Networks: a Survey. ACM Computing Surveys, 2018.



Taxonomy

# Instant Optimal

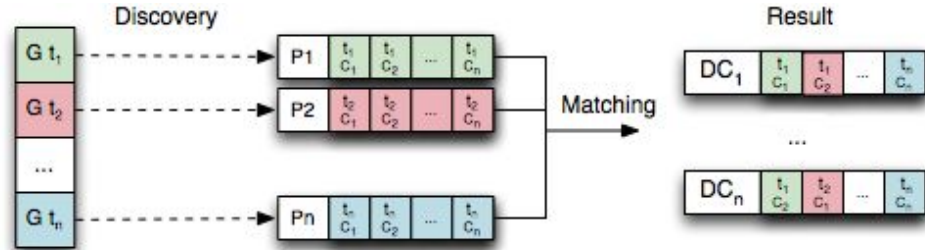
*"Communities found at time  $t$  are optimal for the network at time  $t$ "*

## Strengths

Definition consistent with static CD, parallelisation

## Drawbacks

Lack of smoothness, only Snapshot Network repr.



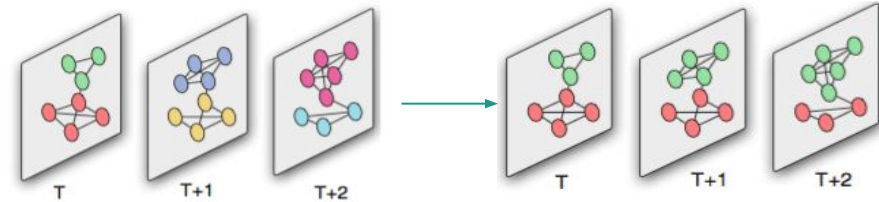
## Taxonomy

# Two-Step

1. Communities are detected at every step using a static algorithm (e.g. Louvain Algorithm)
2. Similarities are computed between communities in consecutive steps (at  $t$  and  $t+1$  (e.g., Jaccard index))

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{|A \cap B|}{|A| + |B| - |A \cap B|}$$

3. Most similar communities are matched between  $t$  and  $t+1$



### Advantages:

- Easy to model, can extend smoothly static approaches

### Drawbacks:

- The reduction to static scenarios through temporal discretization is not always a good idea
  - How to choose the temporal threshold?
  - To what extent can we trust the obtained results?

Taxonomy

# Temporal Trade-Off

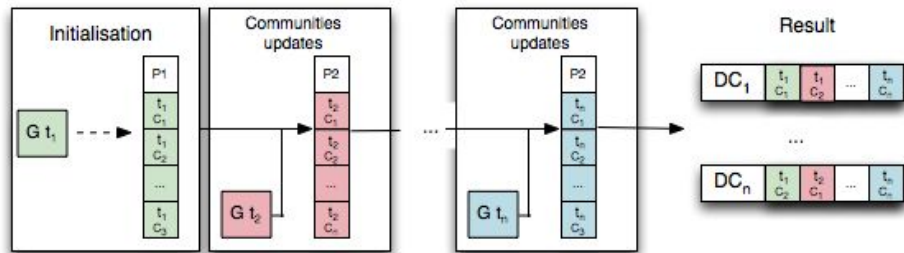
*“Communities found at time  $t$  represent a trade-off between the graph at  $t$  and its previous states”*

## Strengths

Online, incremental, natural smoothness

## Drawback

Iterative, risk of avalanche effect



## Taxonomy

# Tiles

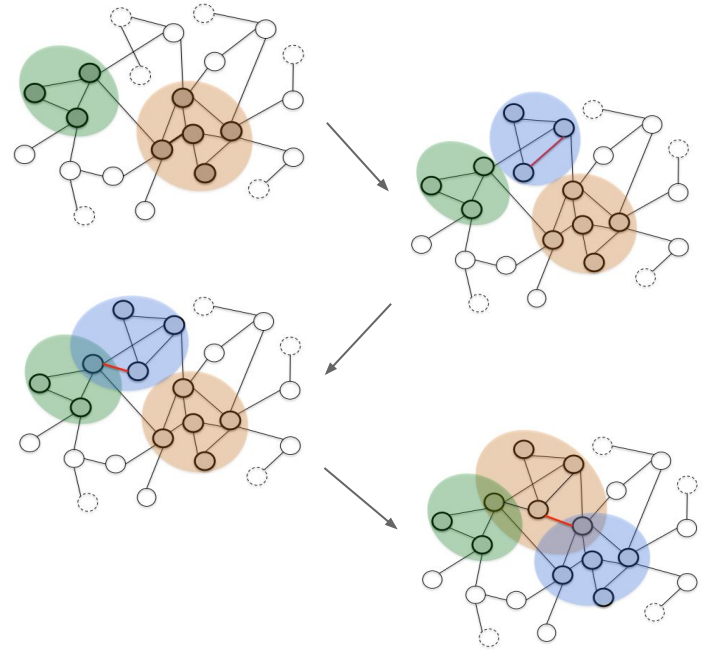
1. Social Interactions define the communities a user belongs to
2. Dynamic graphs as *edge streams*
3. Online updates of communities as nodes/edges appear/vanish

## Advantages:

- Punctual updates of the community structure
- Low computational complexity

## Drawbacks:

- Ad-Hoc model



Rossetti, et al. "Tiles: an online algorithm for community discovery in dynamic social networks." *Machine Learning* 106.8 (2017): 1213-1241.

Taxonomy

# Cross-Time

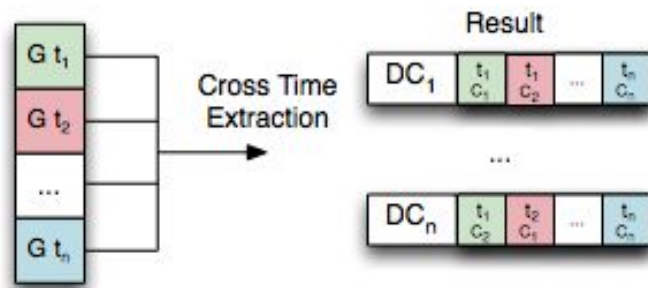
*“Communities at  $t$  are defined relatively to all other steps”*

## Strengths

Perfectly smoothed, stable, solution

## Drawback

Non online, batch computation, lacks incrementality

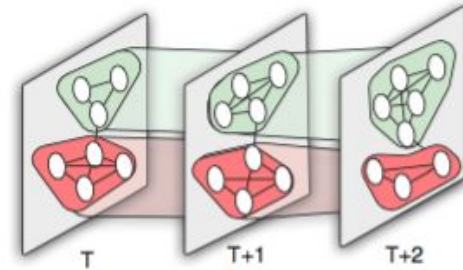


## Taxonomy

# Transversal Network

1. A transversal network is built: nodes are couples (nodes, time), edges link the same node in adjacent snapshots
2. A community detection algorithm is run on this transversal network

(Note: modified Modularity to avoid overestimating expected edges between nodes in different time steps, i.e., custom random graph)



### Advantages:

- Maximal smoothing and stability

### Drawbacks:

- No Community Events are detected
- All the network history needs to be known in advance

Mucha, Peter J., et al. "Community structure in time-dependent, multiscale, and multiplex networks." *science* 328.5980 (2010): 876-878

# Community Detection in Dynamic Networks

Evaluation strategies

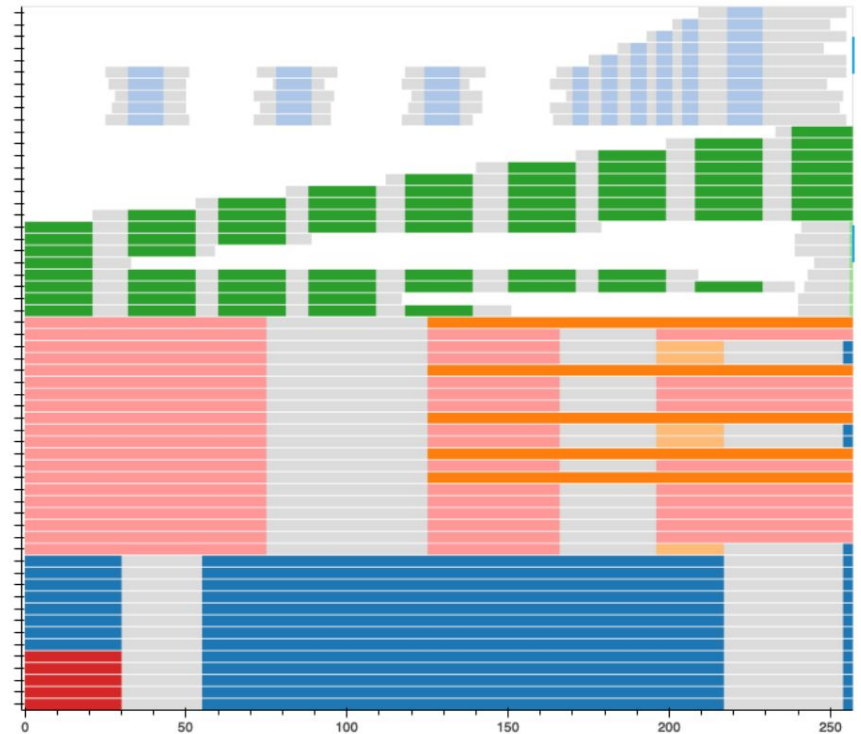
# Strategies

## Internal Evaluation

- Partition quality function  
(i.e., modularity, conductance, density...)
- Community characterization  
(i.e., size distribution, overlap distribution...)
- Execution time and Complexity

## External Evaluation

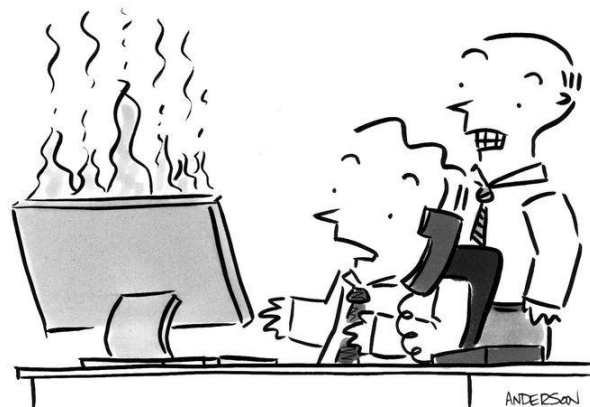
- Ground truth testing  
(or partitions comparison)





# Ground truth testing: Issues

- Few real world datasets with annotated ground truth partition are available (mostly static networks)
- Reliability of partition labelling (semantic partitions not always reflect topological ones)
- Scarcity of network generators handling community dynamics (i.e. birth, death, merge, split)



"I think we're past the point where rebooting will help."

# Summarizing



# Mesoscale Evolutions

Node/edge local dynamics affect community structures

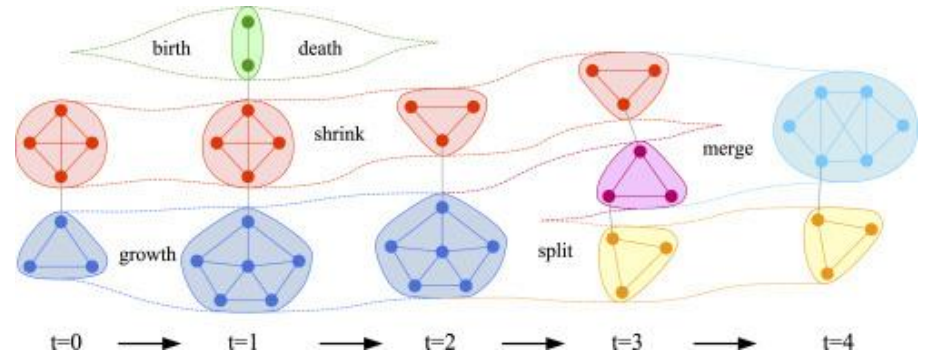
- Communities are subject to events/operations
- Life-cycles can be identified and studied

The complexity behind such ill posed problem grows

- Stability/Persistence
- Smoothness

Every family of approaches depend on

- Specific analytical needs
- Dynamic Network Representation adopted



<https://andreafaila.github.io/teaching/osnam/>