# Link Prediction

Online Social Networks Analysis and Mining

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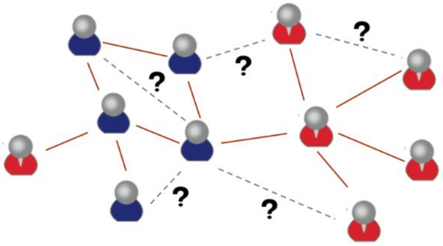
# Link Prediction

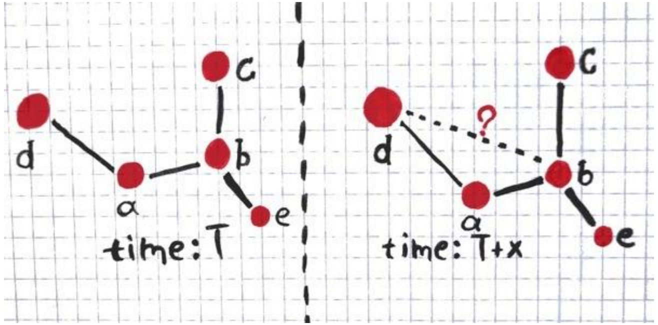
## Goal

Understanding how networks evolve

## Problem definition

Given a snapshot of a network at time t, (accurately) predict the edges that will appear in the network during the interval (t, t+1)





*Liben-Nowell, David, and Jon Kleinberg. "The link-prediction problem for social networks." Journal of the American society for information science and technology 58.7 (2007): 1019-1031.*

Examples of uses of

# Link Prediction

Monitor terrorist networks – deducing possible interactions/missing links between terrorists (without direct evidence)

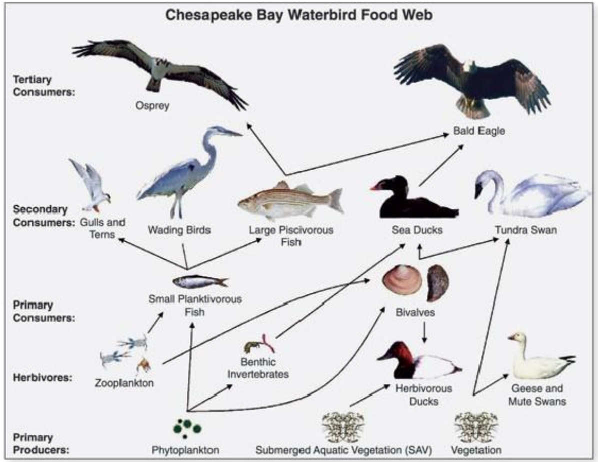
Suggest interactions or collaborations that haven’t yet been exploited within an organization

Friendship prediction (i.e., Facebook)

# Link Prediction

Link prediction is used to predict future possible links in the network (e.g., Facebook).

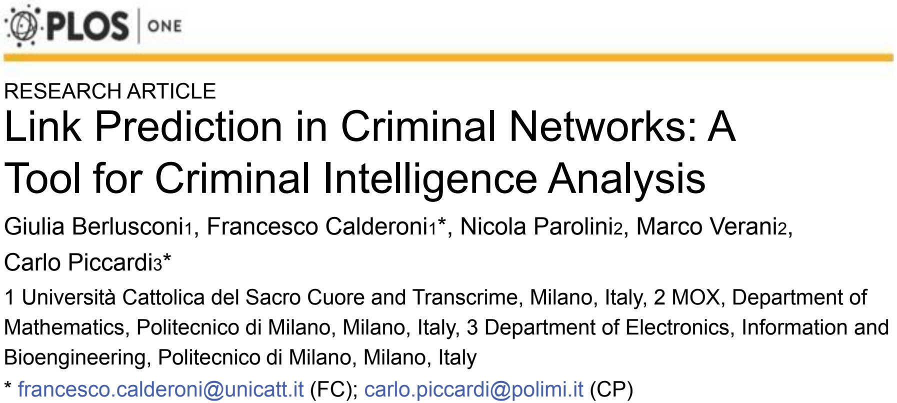
Or, it can be used to predict missing links due to incomplete data (e.g., Food-webs)



facebook

tinder

amazon



PLOS | ONE

RESEARCH ARTICLE

Link Prediction in Criminal Networks: A Tool for Criminal Intelligence Analysis

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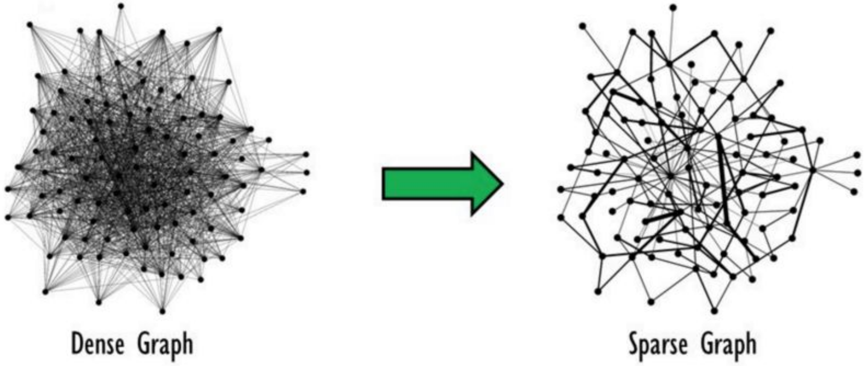
Link Prediction

# Task Complexity

1. Given a graph G = (V,E) the set of possible edges to be predicted is O(|V|^2);

2. Real networks tend to be sparse

**False Positive** prediction issue
(i.e., we are likely to predict edges that will never appear)



# Concretizing an Intuition...

Scientists who are close in the network
(i.e., have common colleagues)

→ will likely collaborate in the future

Goal:

- make this intuitive notion precise and understand which measures of “proximity” leads to accurate predictions

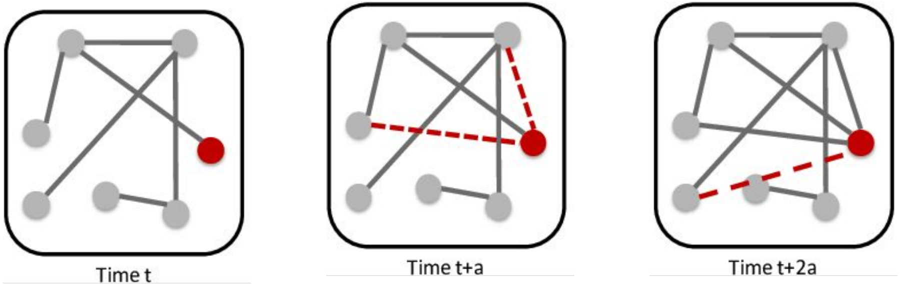
Link Prediction

# Workflow

1. Consider as input a graph G at time t
2. Consider all the possible pairs of nodes (u,v)
3. Compute a link formation score:

   score(u,v)
4. Build a list of all possible edges ordered by scores (from highest to lowest)
5. Verify, following that ordering, the predictions on the graph at time t+1

**score** is a measure of ***proximity***



Time t

Time t+a

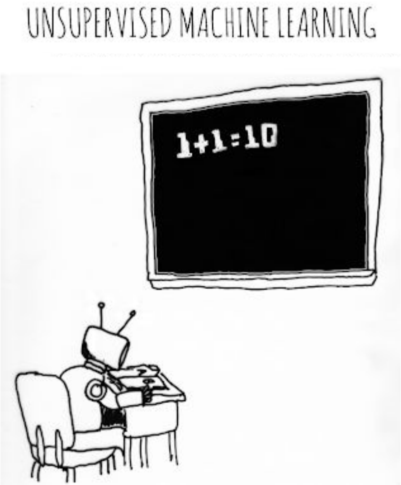
Time t+2a

Link Prediction

# Approaches

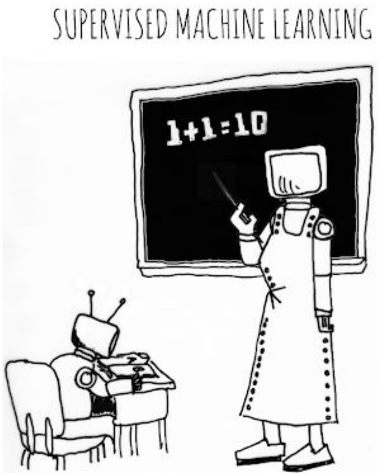
## Unsupervised

Define a set of proximity measures unrelated to the particular network



## Supervised

Extract knowledge from the network in order to improve prediction accuracy



# Unsupervised Link Prediction

# Unsupervised Link Prediction

Unsupervised measurements rely on different structural properties of networks

## Neighborhood measures

- Common Neighbors, Adamic Adar, Jaccard, Preferential Attachment

## Path-based measures

- Graph distance, Katz

## Ranking

- Sim Rank, Hitting time, Page Rank

*Liben -Nowell, David, and Jon Kleinberg. "The link -prediction problem for social networks." Journal of the American society for information science and technology 58.7 (2007): 1019-1031.*

Unsupervised Link Prediction

# Neighborhood measures

*How many friends we have to share in order to become friends?*

Common Neighbors:

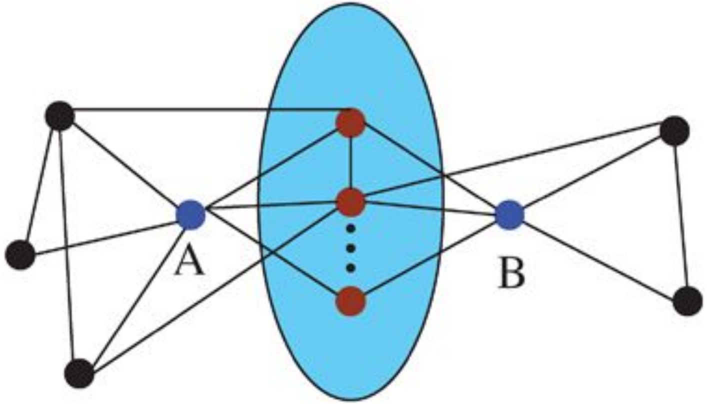
the more friends we share,
the more likely we will become friends

$$\text{score}\ (u, v) = |\Gamma(u) \cap \Gamma(v)|$$

Jaccard:

the more similar our friends circles are,
the more likely we will become friends

$$\text{score}\ (u, v) = \frac{|\Gamma(u) n \Gamma(v)|}{\Gamma(u) \cup r(v)|}$$



Unsupervised Link Prediction

# Neighborhood measures

*How many friends we have to share in order to become friends?*

## Adamic Adar:

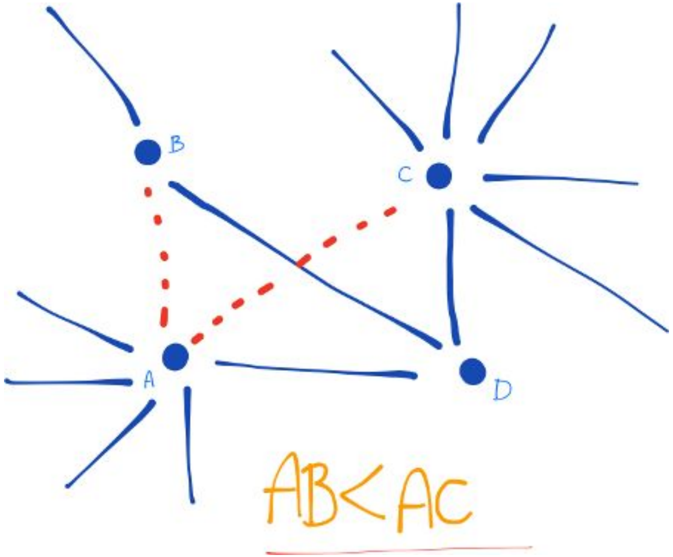
the more selective our mutual friends are,
the more likely we will become friends

$$\text{score}\,(u,v) = \sum_{z\in\Gamma(u)\cap\Gamma(v)} \frac{1}{\log(|\Gamma(z)|)}$$

## Preferential Attachment:

the more friends we have,
the more likely we will become friends

$$\text{score}\,(u,v) = |\Gamma(u)| * |\Gamma(v)|$$



Unsupervised Link Prediction

# Path-based measures

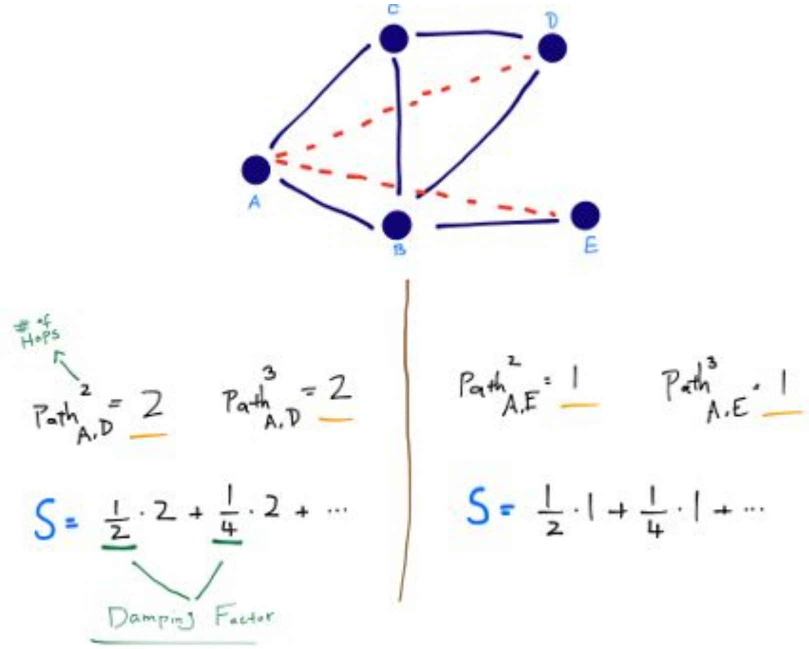
*How distant are we?*

Graph Distance:
(negated) length of the shortest path between two nodes

Katz:
weighted sum over all the paths between two nodes

$$\text{score}\,(u,v) = \sum_{l=1}^{\infty} \beta^{l} |\,\text{paths}\,_{u,v}^{(l)}|$$

where: $paths_{u,v}^{\langle l \rangle}$ ={paths of length exactly $l$ from $u$ to $v$}



C
D
A
B
E
# of Hops
$Path^{2}_{A,D} = 2$
$Path^{3}_{A,D} = 2$
$Path^{2}_{A,E} = 1$
$Path^{3}_{A,E} = 1$
$S = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 2 + \cdots$
$S = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 1 + \cdots$
Damping Factor

Unsupervised Link Prediction

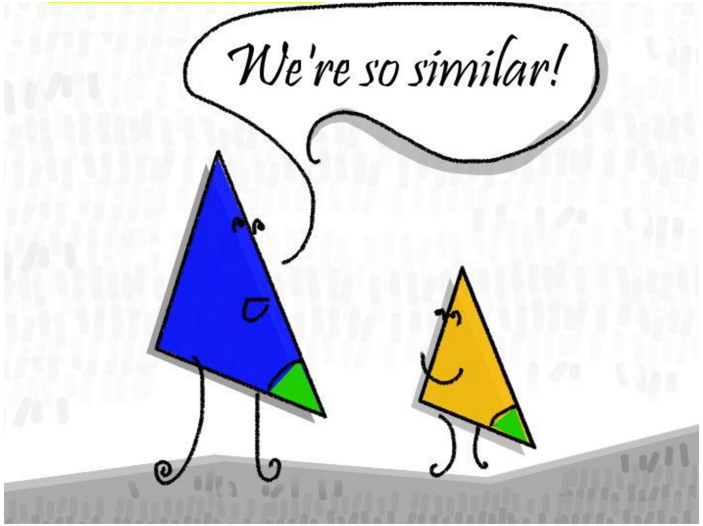
# Ranking

*How similar are we?*

SimRank:
two nodes are *similar* to the extent that their neighborhoods are *similar*

$$\text{similarity}\ (u,v) = \gamma * \frac{\sum_{s\in r(u)}\sum_{n\in\Gamma(v)}\ \text{similarity}\ (a{,}b)}{|\Gamma(u)|*\Gamma(v)|}$$

$$\text{score}\ (u,v) = \ \text{similarity}\ (u,v)$$



Unsupervised Link Prediction

# Limitations

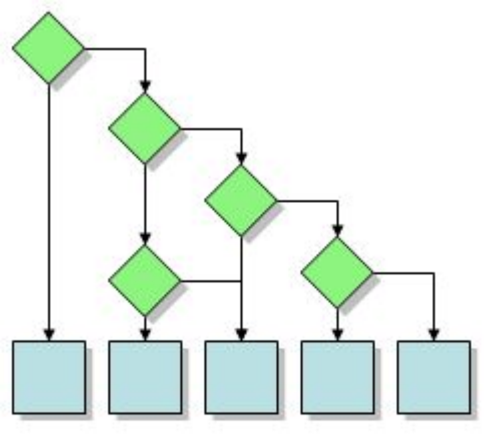
- Different kinds of networks are described by the same general closed formula
- An average overall performance between 6% and 12%

Measure comparison

- No single winner
- Almost all predictors outperform the random predictor
  ⇒ there is useful information in network topology

# Supervised Link Prediction

# Supervised Link Prediction



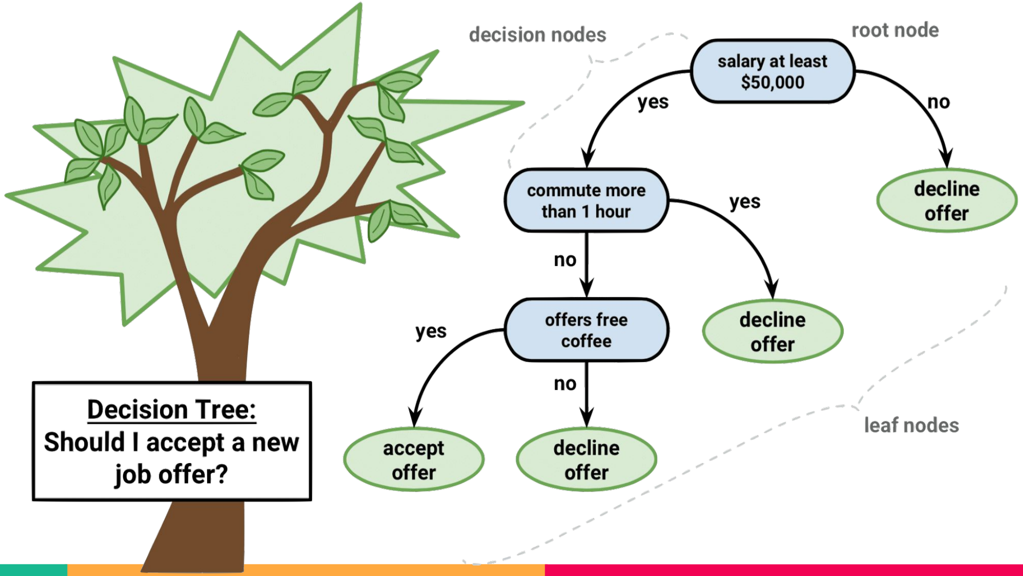
The process is now organized in 4 steps:

1. Split the data in train/test
2. **Learning a model on the train set**
3. **Use the model for prediction**
4. Compare the prediction with the test set

A natural way to do it:
build a "*classifier*" over a set of *network features*.

# Stacking Unsupervised Scores

Learn a Classifier (i.e., a Decision Tree) over unsupervised LP scores to generalize the assumption they made on the network growth model



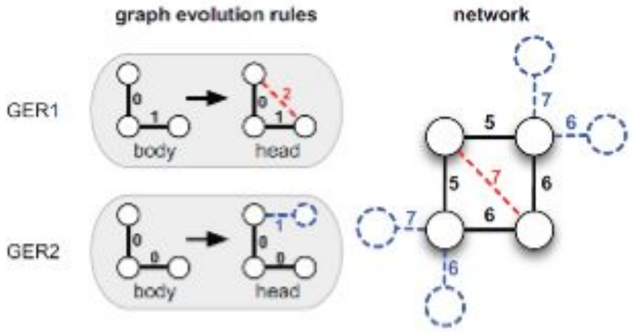
Supervised Link Prediction

# Frequent Pattern Mining

## GERM

Evolution rules can be extracted from the network history and used to identify/predict recurrent patterns

- e.g., generalization of triadic closure



*Berlingerio, Michele, et al. "Mining graph evolution rules." joint European conference on machine learning and knowledge discovery in databases (2009).*

Supervised Link Prediction

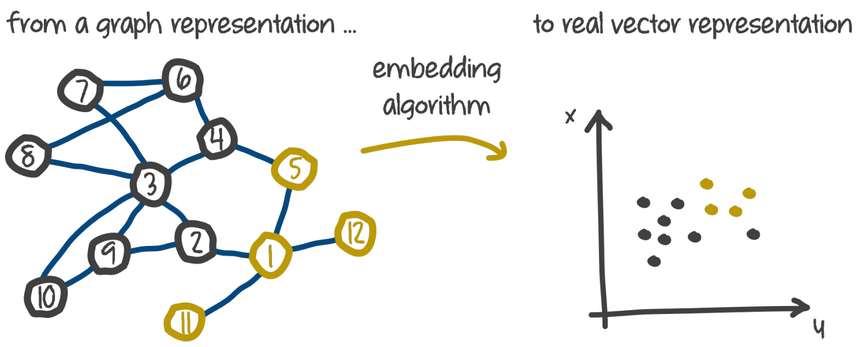
# Network Embedding

Idea

Graphs can be *mapped* into vector spaces

- Node/edge similarity scores can be used to define metric spaces
- Metric spaces enable a more natural application of approaches from DM/ML

NB: Different "mappings" facilitate the solution of different classes of problems


from a graph representation ...
embedding algorithm
to real vector representation
7
6
8
4
5
3
2
9
1
12
10
11
x
y

Supervised Link Prediction

# Limitations

- No Free Lunch
- Model construction is often complex and, usually, more time/resource *demanding* than directly applying unsupervised scores.

Results: Higher performances w.r.t. unsupervised approaches

Embedding is not The Answer,
only a different way to reason on graphs...

# Evaluation

# Evaluation

*Given a predictor p is there a way to decide if it is a "good" one?*

First Step:

verify that p outperforms the *random predictor*.

Random Predictor

each edge has the same probability to appear in the network

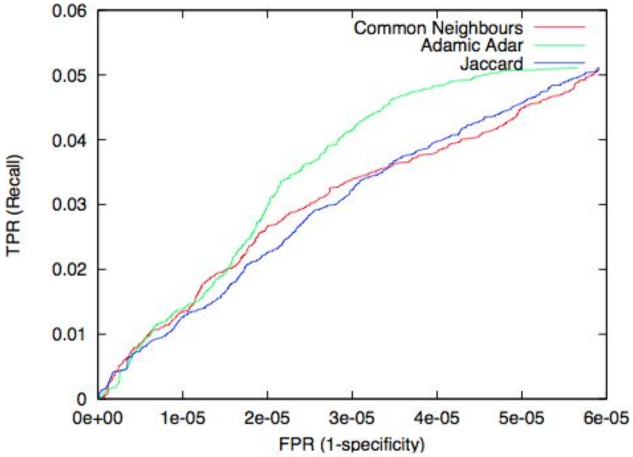
$$\text{performance}\,(p) = \frac{TP}{TP + FP}$$

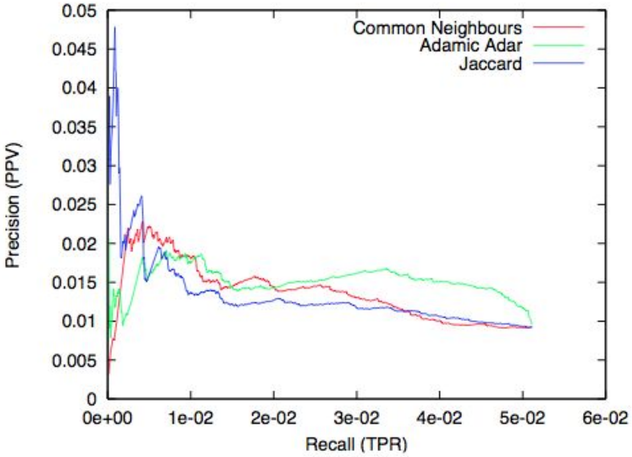
$$\text{ratio} = \frac{\text{performance}\,(p)}{\text{performance}\,(\text{prandom})} = \frac{\text{performance}\,(p)}{\frac{\mathbb{V}(-(\sqrt{V}-1)|}{2} - |E_{\text{old}}|}$$

if ratio > 1 then p is meaningful

# Evaluation: Comparing Predictors

We need to analyze either the performances ratio, ROC and/or Precision Recall curve.





# Evaluation: ROC and PR curve

## Precision Vs. Recall

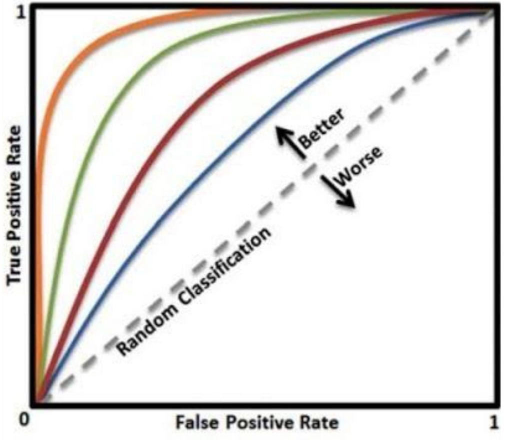
- Precision: PPV = TP/(TP+FP)
- Recall: TPR = TP/(TP+FN)

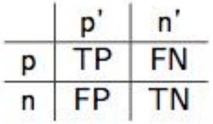
## ROC (Receiver operating characteristic)

- 1-Specificity: FPR = FP/(FP+TN)
- Recall: TPR = TP/(TP+FN)

## Note:

- ROC and PR spaces are isomorphic (the use of ROC is more widespread)
- Numerical comparison can be done using the AUROC (area under the ROC curve)





|   | p' | n' |
|---|---|---|
| p | TP | FN |
| n | FP | TN |

Confusion Matrix

Link Prediction

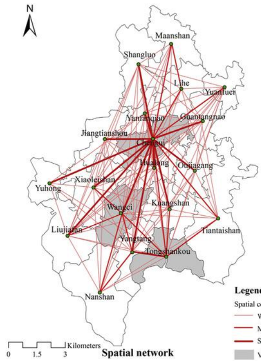
# Something more...

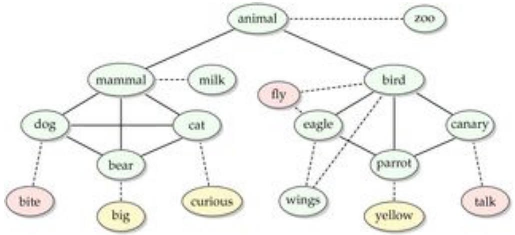
Accuracy could be improved extending simple models with more complex (even semantic) informations:

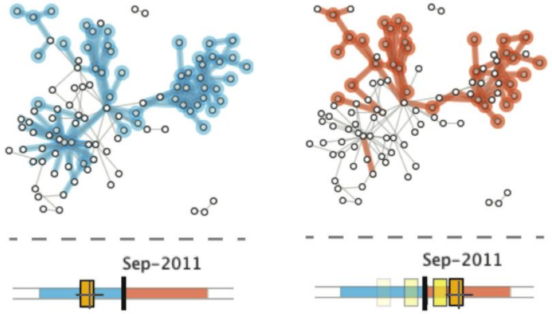
- Link strength
- Geographical information
- ...

Link Prediction needs to be revised while in some scenarios:

- **Dynamic Networks**
- Multiplex networks
- ...







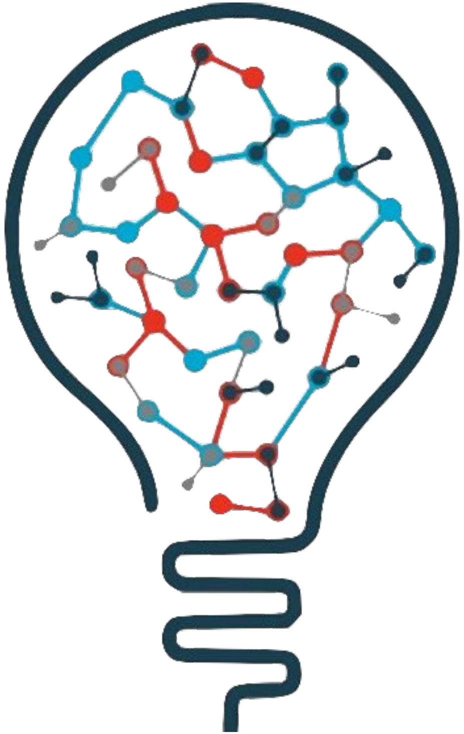
# Key Messages

**Predict new links** that will arise in a network is **not easy**:

1. Networks are, usually, **sparse**
2. **Cold Start** Problem
   - What if I don't have enough information?
     - *Can I predict bridges?*

3. **False Positive** prediction
   - *Bridges* !?!

4. Simple approaches are **"too simple"**
5. Complex approaches are **costly**



Case Study:

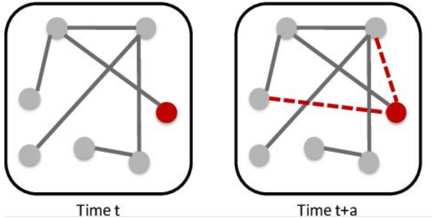
# Interaction Prediction in Dynamic Networks

Case Study

# Interaction Prediction

**Link Prediction** goal:

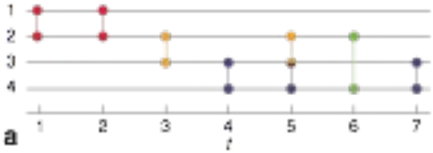
Predict ties that are not present in actual network configuration.



Ties are persistent structures that once appeared cannot disappear (i.e., friendship...)

**Interaction Prediction** goal:

Predict interactions that will occur (either for the first time or not) among nodes already observed in the network.



Interactions are volatile structures that can occur multiple times and whose value can vanish as time goes by (i.e., telephone calls...)

*Rossetti et al. "Interaction prediction in dynamic networks exploiting community discovery." IEEE/ACM ASONAM, 2015.*

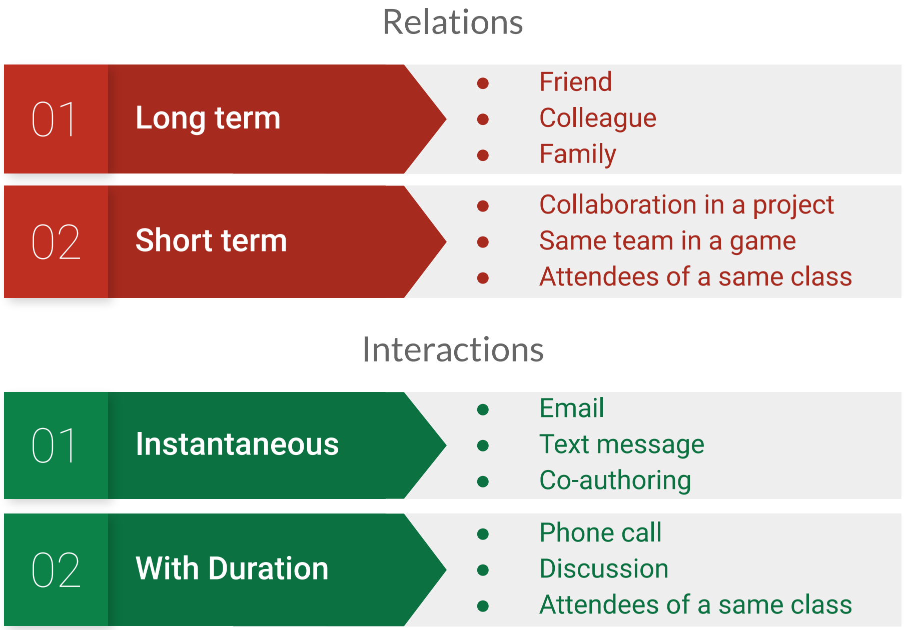
Semantic

# Relations Vs. Interactions

Topological perturbations may have different temporal scales depending on their intrinsic semantic value.

Two families:

- Relations (stable ties)
- Interactions (unstable ties)



Relations

01 Long term

- Friend
- Colleague
- Family

02 Short term

- Collaboration in a project
- Same team in a game
- Attendees of a same class

Interactions

01 Instantaneous

- Email
- Text message
- Co-authoring

02 With Duration

- Phone call
- Discussion
- Attendees of a same class

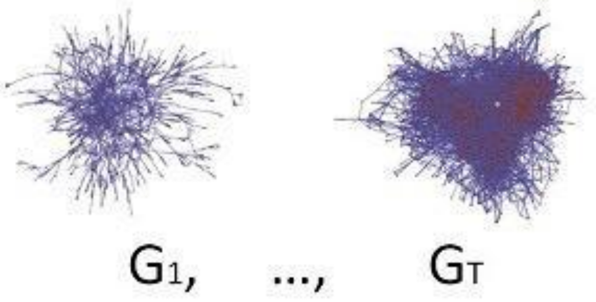
Case Study

# Interaction Prediction

Idea:

- Model network evolution through temporal snapshots;
- False Positive reduction:
  Community Discovery as a bound for strong ties;
- Time-Aware approach:
  time series forecast of topological measures;
- Supervised Approach:
  ensemble of classifiers learnt on the topological features, tested on the forecasted ones.

Given a set $G = \{G_0, \ldots G_t, \ldots G_T\}$ of ordered network observations, with $t \in T = \{0 \ldots T\}$, the **interaction prediction** problem aims to predict new interactions that will took place at time $T + 1$ thus composing $G_{T+1}$.



Case Study

# Interaction Prediction

## Step 1:

For each temporal snapshot t ∈ T compute a partition $C_t = \{C_{t,0}, \ldots, C_{t,k}\}$ of $G_t$ using a community discovery algorithm.

## Step 2:

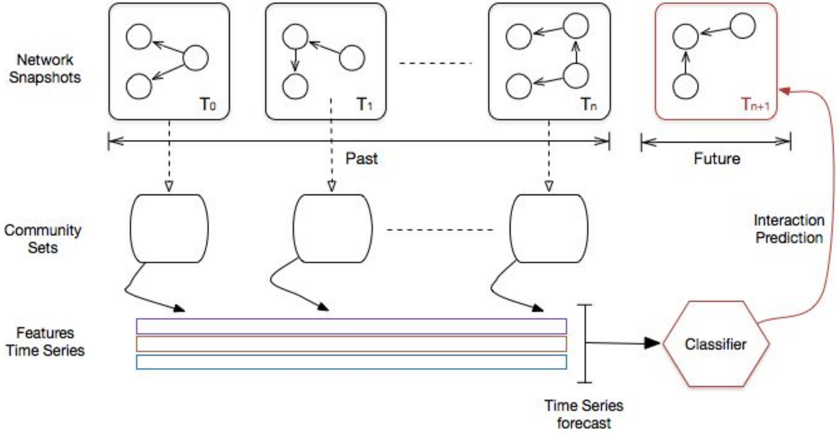
For each t ∈ T compute a set of measures F for each nodes pair (u,v) belonging to at least a community in $C_t$

## Step 3:

For each node pair (u, v) and feature f ∈ F build a time series $S^{u,v}$ and apply a forecasting techniques in order to obtain its future expected value $f^{u,v}$

## Step 4:

Use the set of expected values $f^{u,v}$ to predict future intra-community interactions.



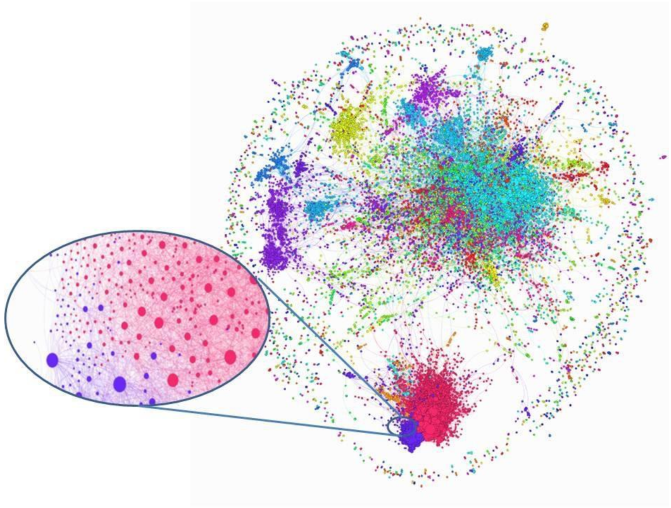
Case Study

# Interaction Prediction

## Step 1: Community Discovery (CD)

Each CD algorithm proposes its own Community Definition.

- **Demon**
  (ego-network based, overlap)
- **Louvain**
  (modularity, crisp partition)
- **Infohiermap**
  (conductance, crisp partition)



Case Study

# Interaction Prediction

## Step 2: Features

On the identified communities we compute three set of features:

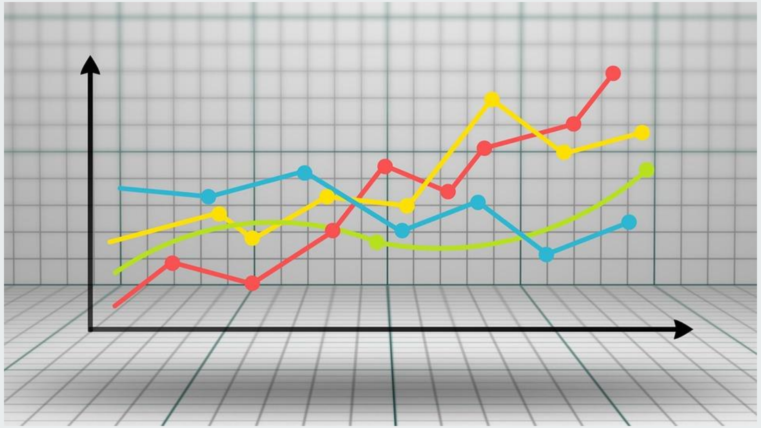
- **Pairwise Structural Features**
  (i.e., Jaccard, CN, Adamic/Adar…)
- **Node Topology Features**
  (PageRank, edge betweenness…)
- **Community Features**
  (i.e., density, size, shared communities, avg. clustering…)

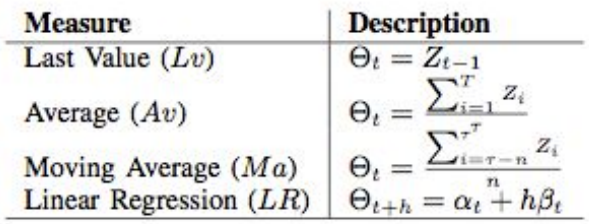
Case Study

# Interaction Prediction

## Step 3: Time Series Forecast

For each time series we apply several forecasting model in order to extract the expected future value.





| Measure | Description |
|---|---|
| Last Value $(Lv)$ | $\Theta_t = Z_{t-1}$ |
| Average $(Av)$ | $\Theta_t = \frac{\sum_{i=1}^{T} Z_i}{}$ |
| Moving Average $(Ma)$ | $\Theta_t = \frac{\sum_{i=\tau-n}^{\tau} Z_i}{n}$ |
| Linear Regression $(LR)$ | $\Theta_{t+h} = \alpha_t + h\beta_t$ |

Case Study

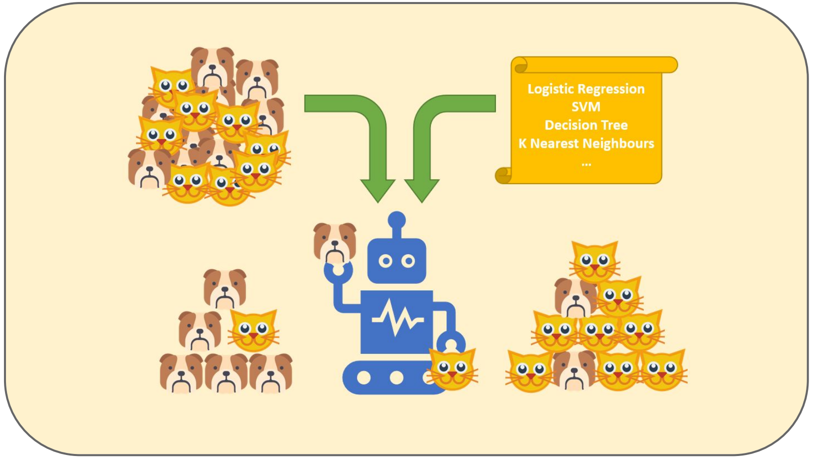
# Interaction Prediction

## Step 4: Classification

Once learned the features we design two different experiments:

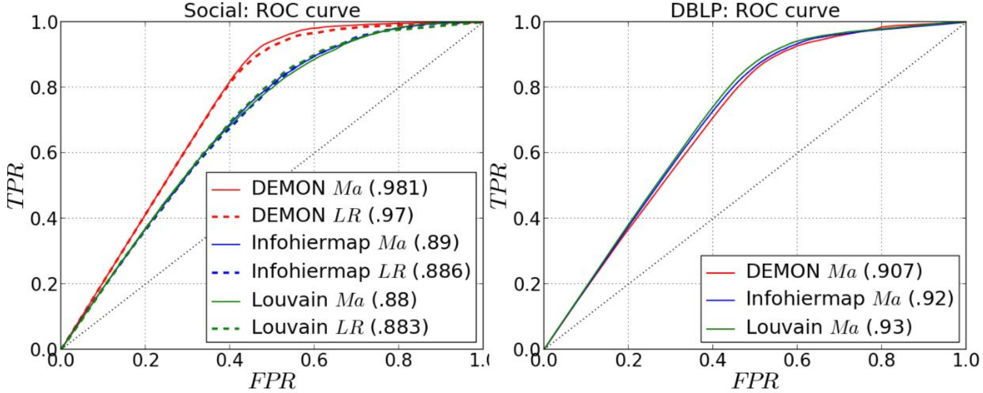
- Balanced Scenario
  The positive and negative class are balanced through downsampling to design a standard baseline

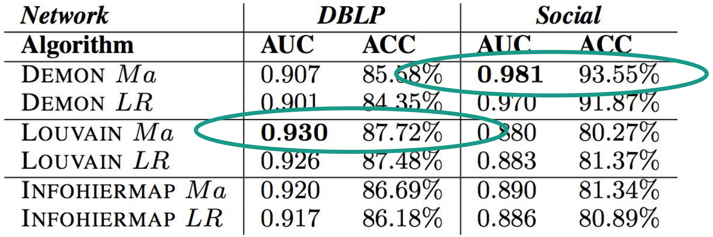
- Unbalanced Scenario
  The data positive/negative class ratio is maintained. Due to network sparsity we observe a strong negative prevalence (~98%)



Case Study

# Interaction Prediction: Balanced Scenario





| *Network* | *DBLP* | | *Social* | |
|---|---|---|---|---|
| **Algorithm** | **AUC** | **ACC** | **AUC** | **ACC** |
| DEMON *Ma* | 0.907 | 85.58% | **0.981** | 93.55% |
| DEMON *LR* | 0.901 | 84.35% | 0.970 | 91.87% |
| LOUVAIN *Ma* | **0.930** | 87.72% | 0.880 | 80.27% |
| LOUVAIN *LR* | 0.926 | 87.48% | 0.883 | 81.37% |
| INFOHIERMAP *Ma* | 0.920 | 86.69% | 0.890 | 81.34% |
| INFOHIERMAP *LR* | 0.917 | 86.18% | 0.886 | 80.89% |

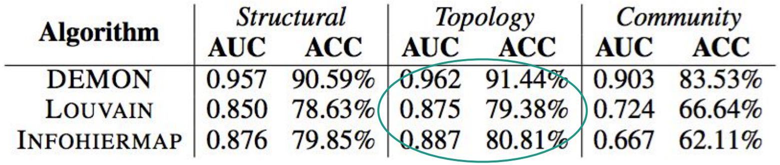
Very high accuracy and AUC

CD approaches contribution to IP is topology sensitive

Case Study

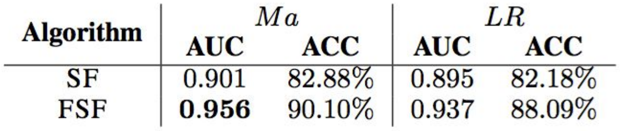
# Interaction Prediction: Balanced Scenario (cont’d)

Which feature set is the most predictive?



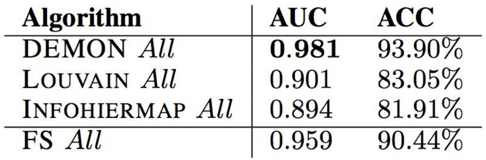
| Algorithm | *Structural* |  | *Topology* |  | *Community* |  |
|---|---|---|---|---|---|---|
|  | **AUC** | **ACC** | **AUC** | **ACC** | **AUC** | **ACC** |
| DEMON | 0.957 | 90.59% | 0.962 | 91.44% | 0.903 | 83.53% |
| LOUVAIN | 0.850 | 78.63% | 0.875 | 79.38% | 0.724 | 66.64% |
| INFOHIERMAP | 0.876 | 79.85% | 0.887 | 80.81% | 0.667 | 62.11% |

False Positive Filtering (FSF)
vs.
No Filtering (SF)



| Algorithm | *Ma* |  | *LR* |  |
|---|---|---|---|---|
|  | **AUC** | **ACC** | **AUC** | **ACC** |
| SF | 0.901 | 82.88% | 0.895 | 82.18% |
| FSF | **0.956** | 90.10% | 0.937 | 88.09% |

All Forecast with Filtering
vs.
No Filtering



| **Algorithm** | **AUC** | **ACC** |
|---|---|---|
| DEMON *All* | **0.981** | 93.90% |
| LOUVAIN *All* | 0.901 | 83.05% |
| INFOHIERMAP *All* | 0.894 | 81.91% |
| FS *All* | 0.959 | 90.44% |

Case Study

# Interaction Prediction: Unbalanced Scenario

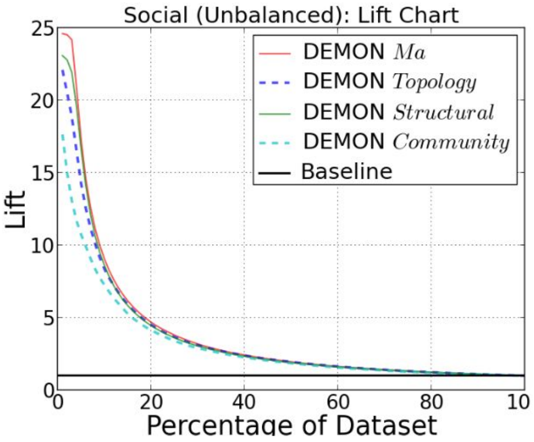
Negative class:

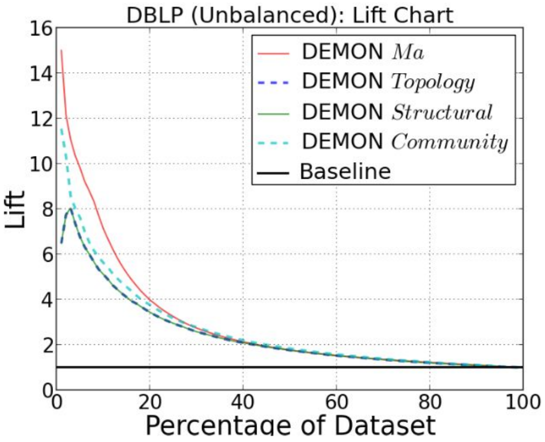
- Social 95.9%
- DBLP 98.9%

**Very hard baselines**

- majority classifiers scores ~.96 and ~.99 precision
  (always predicting “no edges”)

- the proposed workflow is able to reach ~.96 and ~.45 precision w.r.t. the positive class


Social (Unbalanced): Lift Chart


DBLP (Unbalanced): Lift Chart

Case Study

# Interaction Prediction: What about weak links?

High accuracy is guaranteed by focusing the prediction on intra-community interactions.

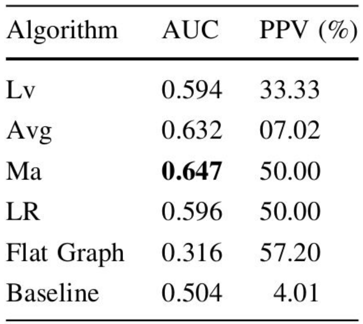
### Inter-Community Interaction Prediction

Focus on the predicting the presence/absence of *at least* a new interaction across two communities

- no identification of the “real” endpoints
- no identification of the multiplicity

### Idea

1. Construct a new network where the meta-nodes are the communities
2. Apply the same workflow to such graph



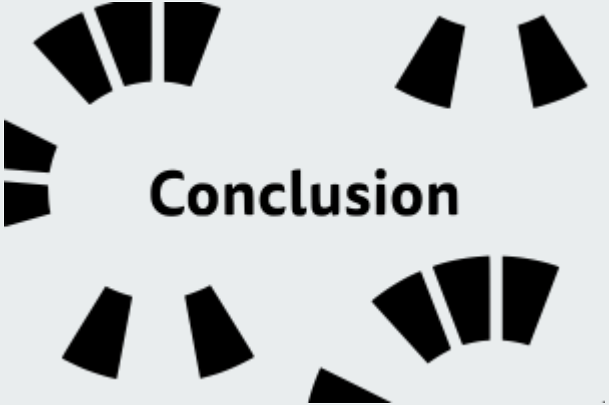
| Algorithm | AUC | PPV (%) |
|---|---|---|
| Lv | 0.594 | 33.33 |
| Avg | 0.632 | 07.02 |
| Ma | **0.647** | 50.00 |
| LR | 0.596 | 50.00 |
| Flat Graph | 0.316 | 57.20 |
| Baseline | 0.504 | 4.01 |

Infohiermap performances for the inter-community prediction. Like in the balanced scenario, the Moving Average Ma forecasted features allow for the best classification models

In bold the AUC of the best performing approach

Case Study

# Interaction Prediction



Even though Interaction prediction is a complex problem it is possible to reach high accuracy through:

- Target selection:
  False Positive reduction via Community Discovery
  Weak interactions treated as "*special cases*"
- Local topology history analysis:
  Feature forecast via Time Series analysis

Moreover, each type of datasets demands a specific CD algorithm:

- One-to-one interactions (i.e., social ones)
- Many-to-many interactions (i.e., co-authorship relations)